UE Fluids   Problem F11 Solution   S '08

a) Assuming low speed flow, $\rho_1 \simeq \rho_2 \simeq \rho \rightarrow$ Bernoulli valid.

$$\therefore\ p_2 + \tfrac{1}{2}\rho V_2^2 = p_1 + \tfrac{1}{2}\rho V_1^2 \rightarrow \boxed{p_2 - p_1 = \tfrac{1}{2}\rho\left(V_1^2 - V_2^2\right)}$$

Using conditions at point 2: $\rho = \rho_2 = \dfrac{p_2}{RT_2} = \dfrac{10^5\,Pa}{287\,J/kgK \cdot 300K} = 1.161\ kg/m^3$

$$\therefore\ \boxed{p_2 - p_1 = \tfrac{1}{2}\cdot 1.161\left(10^2 - 40^2\right) = -871.08\ Pa}$$

b) $\dfrac{p_1}{p_2} = \dfrac{p_2 - (p_2 - p_1)}{p_2} = \dfrac{100871.1}{100000} = 1.00871$, $\boxed{p_1 = 100871.1\ Pa}$

$\dfrac{\rho_1}{\rho_2} = \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 1.006214$, $\boxed{\rho_1 = \rho_2 \cdot 1.006214 = 1.1682\ kg/m^3}$

$\dfrac{T_1}{T_2} = \left(\dfrac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = 1.00248$, $\boxed{T_1 = T_2 \cdot 1.00248 = 300.74\ K}$

c) $\Delta\rho/\rho = 0.00621$ i.e. 0.62% change, very small

→ so assuming $\rho \approx$ constant is reasonable

d) The mass in a soap bubble cannot change.

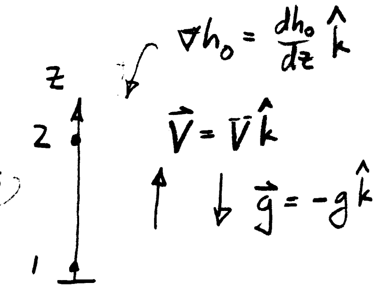
$m_1 = m_2 \rightarrow \rho_1 \mathcal{V}_1 = \rho_2 \mathcal{V}_2$, $\dfrac{\mathcal{V}_2}{\mathcal{V}_1} = \boxed{\dfrac{\rho_1}{\rho_2} = 1.00621}$

The soap bubble expands by 0.00621, or $\boxed{+0.62\%}$

UE Fluids Problem F12 Solution S'08

a) Total enthalpy eq'n: $\frac{Dh_o}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} + \dot{q} + \vec{g}\cdot\vec{V}$

Steady, adiabatic: $\frac{\partial p}{\partial t} = 0$, $\dot{q} = 0$, $\frac{\partial h_o}{\partial t} = 0$



$\therefore \frac{Dh_o}{Dt} \equiv \cancelto{0}{\frac{\partial h_o}{\partial t}} + \vec{V}\cdot\nabla h_o = \vec{g}\cdot\vec{V}$ evaluate dot products

$\vec{V}\cdot\nabla h_o = V\frac{dh_o}{dz}$, $\vec{g}\cdot\vec{V} = -gV$

so $V\frac{dh_o}{dz} = -gV$ cancel V & integrate...

$h_o = -gz + c \rightarrow \boxed{h_{o_2} - h_{o_1} = -g(z_2 - z_1)} = -9810\ \mathrm{m^2/s^2}$ 1000 m

$h_{o_2} = h_2 + \frac{1}{2}V_2^2$, $h_{o_1} = h_1 + \frac{1}{2}V_1^2$

$h_2 - h_1 = h_{o_2} - h_{o_1} - \frac{1}{2}\cancelto{0}{(V_2^2 - V_1^2)}$ since $V_2 = V_1 = V$

$$\boxed{T_2 - T_1 = (h_2 - h_1)/c_p = -9810\ \mathrm{m^2/s^2} / 1004\ \mathrm{J/kg\,K} = -9.77\ \mathrm{K}}$$

b) The flow is adiabatic ($\dot{q} = 0$), and reversible (no friction).

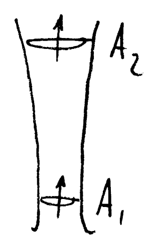
$\therefore$ flow is isentropic.

c) Since flow is isentropic, $\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$

$$\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_1 + (T_2 - T_1)}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{300 - 9.77}{300}\right)^{\frac{1}{\gamma-1}} = 0.9206}$$

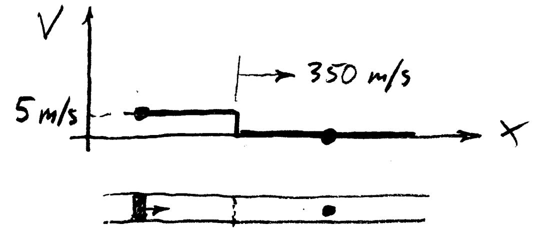
Constant mass flow: $\rho_2 V_2 A_2 = \rho_1 V_1 A_1 \rightarrow \rho_2 A_2 = \rho_1 A_1$ since $V_2 = V_1 = V$

$$\boxed{\frac{A_2}{A_1} = \frac{\rho_1}{\rho_2} = 1.0863}$$



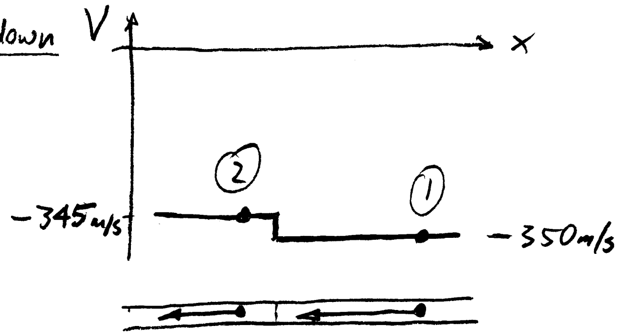
UE Fluids F13 Solution S '08

a) In tube's frame, air on right is still, Air on left of shock moves with piston at 5 m/s



In shock frame, velocities are shifted down by shock speed $V_s = 350$ m/s

$$V_{\text{shock-frame}} = V_{\text{tubeframe}} - V_s$$



b) $h_{o_1} \equiv h_1 + \frac{1}{2}V_1^2$, $\quad h_1 = c_p T_1 = 1004\ \text{J/kg K} \cdot 300\text{K} = 301200\ \text{m}^2/\text{s}^2$

$$\tfrac{1}{2}V_1^2 = \tfrac{1}{2}(-350\ \text{m/s})^2 = 61250\ \text{m}^2/\text{s}^2$$

$\therefore \boxed{h_{o_1} = 301200 + 61250 = 362450\ \text{m}^2/\text{s}^2}$

In shock frame the flow is steady (and adiabatic) so $h_o = ¢$

$\therefore \boxed{h_{o_2} = h_{o_1} = 362450\ \text{m}^2/\text{s}^2}$

c) $\boxed{h_2 = h_{o_2} - \tfrac{1}{2}V_2^2 = 362450\ \text{m}^2/\text{s} - \tfrac{1}{2}(-345\ \text{m/s})^2 = 302937.5\ \text{m}^2/\text{s}^2}$

$\boxed{T_2 = h_2/c_p = 301.73\ \text{K}}$

d) This is weak shock because all the changes across the shock are small percentagewise, e.g. $|V_2 - V_1|/V_s \ll 1$, $|h_2 - h_1|/h_1 \ll 1$ etc.

e) Assuming isentropic relations are valid,

$$p_2 = p_1 (T_2/T_1)^{\frac{\gamma}{\gamma-1}} = 10^5\ \text{Pa} \cdot (301.73/300)^{3.5} = 10203\ \text{Pa}$$

$\boxed{p_2 - p_1 = 10203 - 10000 = 203\ \text{Pa}}$